One of the questions about which humanity has often wondered is the arrow of time. Why does temporal evolution seem irreversible? That is, we often see objects break into pieces, but we never see them reconstitute spontaneously. This observation was first put into scientific terms by the so-called second law of thermodynamics: entropy never decreases. However, this law does not explain the origin of irreversibility; it only quantifies it. Kinetic theory gives a consistent explanation of irreversibility based on a statistical description of the motion of electrons, atoms, and molecules. The concepts of kinetic theory have been applied to innumerable situations including electronics, the production of particles in the early universe, the dynamics of astrophysical plasmas, granular avalanches, or the motion of small microorganisms in water, with excellent quantitative agreement. This book presents the fundamentals of kinetic theory, considering classical paradigmatic examples (classical and quantum gases, plasmas, Brownian particles, and electronic transport) together with modern applications and numerical methods. The text is balanced between exploring the fundamental concepts of kinetic theory (irreversibility, transport processes, separation of time scales, conservation, coarse graining, distribution functions, etc.) and the results and predictions of the theory, where the relevant properties of different systems are computed.

Rodrigo Soto is Full Professor at the Physics Department, FCFM, Universidad de Chile, Santiago de Chile.

'Soto has written the modern, accessible textbook that this field of kinetic theory deserves. Written by a practitioner, in an informal style with clear motivations, lots of figures, frequent references to further reading and pointers to the minefields awaiting the keen novice, makes it particularly useful for anyone who wants to get their hands dirty quickly.'
Gunnar Pruessner, Imperial College London, UK

'Professor Soto has done a marvellous job of producing an introductory text on non-equilibrium systems and transport phenomena. This book, written in an unassuming informal style with a wide range of exercises, is suitable for advanced undergraduates or early graduate students. The organisation, beginning with fundamental concepts and methods, and proceeding to classical and modern applications, will spark interest in many areas of physics.'
Dimitri Vvedensky, Imperial College London, UK
# Contents

1 Basic concepts  
1.1 Velocity distribution function  
1.2 The Maxwell–Boltzmann distribution function  
1.3 Densities and fluxes  
  1.3.1 Stress tensor and energy flux  
  1.3.2 Stress tensor and heat flux in equilibrium  
  1.3.3 Flux distribution  
1.4 Collision frequency  
1.5 Mean free path  
1.6 Transport properties in the mean free path approximation  
  1.6.1 Thermal conductivity  
  1.6.2 Viscosity  
  1.6.3 Wall slip  
  1.6.4 Self-diffusion  
1.7 Drude model for electric transport  
Exercises  

2 Distribution functions  
2.1 Introduction  
2.2 Hamiltonian dynamics  
2.3 Statistical description of the phase space  
2.4 Equilibrium distribution  
2.5 Reduced distributions  
2.6 Microscopic and average observables  
  2.6.1 Global observables  
  2.6.2 Densities  
  2.6.3 Fluxes  
  2.6.4 Conservation equations  
2.7 BBGKY hierarchy  
  2.7.1 Equation for the one-particle distribution  
2.8 Generalisation to mixtures  
2.9 Reduced distributions in equilibrium and the pair distribution function  
2.10 Master equations  
2.11 Application: systems with overdamped dynamics  
Further reading  
Exercises
3 The Lorentz model for the classical transport of charges 39
  3.1 Hypothesis of the model 39
  3.2 Lorentz kinetic equation 41
  3.3 Ion distribution function 42
  3.4 Equilibrium solution 43
  3.5 Conservation laws and the collisional invariants 43
  3.6 Kinetic collision models 44
    3.6.1 Rigid hard spheres 45
    3.6.2 Thermalising ions: the BGK model 46
  3.7 Electrical conduction 46
    3.7.1 Conservation equation 46
    3.7.2 Linear response 47
    3.7.3 Ohm’s law 47
    3.7.4 Electrical conductivity 47
    3.7.5 Frequency response 50
  3.8 Relaxation dynamics 50
    3.8.1 Properties of the linear operator 51
    3.8.2 Kinetic gap 52
    3.8.3 Spectrum of the linear operator 53
    3.8.4 Diffusive behaviour 54
    3.8.5 Rigid hard spheres 54
    3.8.6 Time scales 55
  3.9 The Chapman–Enskog method 56
  3.10 Application: bacterial suspensions, run-and-tumble motion 58
Further reading 60
Exercises 61

4 The Boltzmann equation for dilute gases 63
  4.1 Formulation of the Boltzmann model 63
    4.1.1 Hypothesis 63
    4.1.2 Kinematics of binary collisions 64
  4.2 Boltzmann kinetic equation 66
    4.2.1 General case 66
    4.2.2 Hard sphere model 67
  4.3 General properties 68
    4.3.1 Balance equations and collisional invariants 68
    4.3.2 $H$-theorem 70
    4.3.3 On the irreversibility problem 73
  4.4 Dynamics close to equilibrium 74
    4.4.1 Linear Boltzmann operator 74
    4.4.2 Spectrum of the linear Boltzmann equation 75
    4.4.3 Time scales 77
  4.5 BGK model 77
  4.6 Boundary conditions 79
  4.7 Hydrodynamic regime 79
    4.7.1 The hydrodynamic equations 79
    4.7.2 Linear response 81
    4.7.3 Variational principle 82
4.7.4 The Chapman–Enskog method 82
4.8 Dense gases 86
  4.8.1 The Enskog model for hard sphere gases 86
  4.8.2 Virial expansion 88
4.9 Application: granular gases 89
4.10 Application: the expanding universe 91
Further reading 92
Exercises 92

5 Brownian motion 95
  5.1 The Brownian phenomenon 95
  5.2 Derivation of the Fokker–Planck equation 96
  5.3 Equilibrium solutions 98
    5.3.1 Homogeneous equilibrium solution and the
         fluctuation–dissipation relation 98
    5.3.2 Equilibrium solution under external potentials 99
  5.4 Mobility under external fields 101
  5.5 Long-time dynamics: diffusion 102
    5.5.1 Solution of the diffusion equation 102
    5.5.2 Green–Kubo expression 104
    5.5.3 Coarse-grained master equation 105
    5.5.4 Eigenvalue analysis 106
    5.5.5 Chapman–Enskog method 107
    5.5.6 Boundary conditions 108
  5.6 Early relaxation 108
  5.7 Rotational diffusion 109
  5.8 Application: light diffusion 110
  5.9 Application: bacterial alignment 111
Further reading 112
Exercises 113

6 Plasmas and self-gravitating systems 115
  6.1 Long-range interactions 115
  6.2 Neutral plasmas 116
    6.2.1 Introduction 116
    6.2.2 Debye screening 117
    6.2.3 Vlasov equation 119
    6.2.4 Stationary solutions 122
    6.2.5 Dynamical response 122
  6.3 Waves and instabilities in plasmas 125
    6.3.1 Plasma waves 125
    6.3.2 Landau damping 126
    6.3.3 Instabilities 130
  6.4 Electromagnetic effects 131
    6.4.1 Magnetic fields 131
    6.4.2 Hydrodynamic equations 131
  6.5 Self-gravitating systems 132
    6.5.1 Kinetic equation 132
6.5.2 Self-consistent equilibrium solutions 133
6.5.3 Jeans instability 135
6.6 Beyond mean field 135
  6.6.1 Velocity relaxation and dynamical friction 135
  6.6.2 Slow relaxation 136
  6.6.3 Kinetic equations 137
6.7 Application: point vortices in two dimensions 138
Further reading 140
Exercises 140

7 Quantum gases 143
  7.1 Boson and fermion ideal gases at equilibrium 143
    7.1.1 Description of the quantum state 143
    7.1.2 Equilibrium distributions 145
  7.2 Einstein coefficients 146
  7.3 Scattering transition rates 148
  7.4 Master kinetic equation 149
  7.5 Equilibrium solutions 151
  7.6 Where is the molecular chaos hypothesis? 152
  7.7 Phonons 153
    7.7.1 Ideal gas of phonons 153
    7.7.2 Phonon–phonon interactions 155
    7.7.3 Phonon–electron interactions 160
  7.8 Application: lasers 161
  7.9 Application: quark–gluon plasma 164
Further reading 166
Exercises 166

8 Quantum electronic transport in solids 169
  8.1 Electronic structure 169
  8.2 Fermi–Dirac distribution, conductors, and insulators 169
  8.3 Boltzmann–Lorentz equation 171
    8.3.1 Distribution function 171
    8.3.2 Scattering processes 172
    8.3.3 Semiclassical kinetic equation 173
    8.3.4 Linear collision operator 174
  8.4 Time-independent point defects 175
    8.4.1 Transition rates 175
    8.4.2 Spherical models 176
  8.5 Relaxation time approximation 177
  8.6 Electrical conductivity 177
    8.6.1 Qualitative description: metals and insulators 177
    8.6.2 Conductivity of metals 179
    8.6.3 Finite-temperature effects 181
    8.6.4 Electron–phonon interactions 182
    8.6.5 Multiple scattering mechanisms and the Matthiessen rule 184
  8.7 Thermal conductivity and Onsager relations 185
8.7.1 Wiedemann–Franz law 188
8.8 Transport under magnetic fields 189
  8.8.1 Equilibrium solution 190
  8.8.2 Linear response to electric fields 190
  8.8.3 Hall effect and the magnetoresistance 191
8.9 Thomas–Fermi screening 192
8.10 Application: graphene 193
Further reading 196
Exercises 196

9 Semiconductors and interband transitions 199
  9.1 Charge carriers: electrons and holes 199
  9.2 Doped materials and extrinsic semiconductors 200
  9.3 Kinetic equation 202
    9.3.1 Generation–recombination 203
  9.4 Hydrodynamic approximation 204
  9.5 Photoconductivity 204
  9.6 Application: the diode or p–n junction 205
Further reading 207
Exercises 207

10 Numerical and semianalytical methods 209
  10.1 Direct approach 209
  10.2 Method of moments 209
    10.2.1 Local equilibrium moment method 211
    10.2.2 Grad’s method 211
  10.3 Particle-based methods 212
    10.3.1 Sampling 212
    10.3.2 Random numbers 213
    10.3.3 Streaming motion 214
    10.3.4 Brownian motion 214
    10.3.5 Long-range forces 216
    10.3.6 Collisions 219
    10.3.7 Quantum effects 221
    10.3.8 Boundary conditions 222
Further reading 222
Exercises 223

A Mathematical complements 225
  A.1 Fourier transform 225
  A.2 Dirac delta distributions 225
  A.3 Eigenvalues of a perturbed operator 227
    A.3.1 Statement of the problem 227
    A.3.2 Order $\mathcal{O}(\varepsilon^0)$ 227
    A.3.3 Order $\mathcal{O}(\varepsilon^1)$ 227
Exercises 229

B Tensor analysis 230
  B.1 Basic definitions 230
B.2  Isotropic tensors 232
B.3  Tensor products, contractions, and Einstein notation 233
B.4  Differential operators 234
B.5  Physical laws 234
Exercises 235

C  Scattering processes 236
C.1  Classical mechanics 236
   C.1.1  Kinematics of binary collisions 236
   C.1.2  Geometrical parameterisation 237
   C.1.3  Scattering for hard sphere, Coulomb, and gravitational potentials 237
C.2  Quantum mechanics 238
   C.2.1  Time-dependent perturbation theory 238
   C.2.2  Fermi golden rule 239
Exercises 240

D  Electronic structure in crystalline solids 242
D.1  Crystalline solids 242
D.2  Band structure 242
   D.2.1  Bloch theorem 243
   D.2.2  Energy bands 245
   D.2.3  Bloch velocity and crystal momentum equation 245
   D.2.4  Self-consistent potential 246
D.3  Density of states 247
   D.3.1  Free electron gas 247
   D.3.2  General case in three dimensions 248
Exercises 249

References 250

Index 255